Game theory presented to Italian high school students in connection with infinity computing

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Abstract. Game Theory is a rather vast discipline, the purpose of which is to analyze the strategic behaviors of decision-makers (players), or to study the situations in which different players interact pursuing common, different or conflicting objectives. The first purpose of this paper is to discuss the possibility of approaching elementary game theory in high schools. The second objective is to analyze the students' response in terms of learning, but also of liking. Is it possible *"giocare con la teoria dei giochi"* (Engl. transl: play with game theory) in the classroom? The third objective is to highlight the possible links between elementary game theory and the *grossone*-based system introduced by Y.D. Sergeyev. Once again with the didactic aspect of providing high school students with an "easy" and stimulating approach to modern research fields in mathematics. In particular, we describe the approach to a little cycle of lessons of two classes and the performances of a conclusive students' class test. We will see a great students response, above all regarding the connection between game theory and the grossone system.

Keywords: Game theory · Strategic game representation · Mathematical education · Infinity computing · Grossone

1 Introduction

Game Theory is a rather vast discipline, the purpose of which is to analyze the strategic behavior of decision makers (players), that is to study the situations in which different players interact pursuing common, different or conflicting objectives. Players are hypothesized to have rational and intelligent behaviors.

A game is defined as *a situation of strategic interaction between at least two players who behave rationally and intelligently on the basis of rules known to all.*

The term *rational* is directly connected to the neoclassical concept of rationality meaning that each player tries to maximize his final outcome, given a utility function that establishes an order of preference. However, this assumption is however superseded in the sense that it is required that the players, not only are able to solve problems optimally subjected to constraints given, but are aware that their choices influence the behavior of other players.

For this reason, it is preferred to connote the players as *intelligent* and not only as *rational*, in order to indicate the ability both to predict and take into account the behavior of the other players. Game theory can have two different roles:

- The first one (the positive role) is to interpret reality explaining why in certain conflictual situations, the subjects involved (players) adopt certain strategies and certain tactics;
- the second (the prescriptive role) is that one of determining which situations of equilibrium can (or cannot) occur as a result of the interaction of the decision-makers.

In any case, the concepts of solution which are used in the game theory intend to describe the strategies, which the decision-makers should follow, individually or jointly, as a consequence of the rationality hypotheses mentioned above. If then in reality the decision-makers deviate from what is foreseen by the theory, it is, undoubtedly, necessary to ask whether this happens because the model does not capture all the relevant aspects of a situation, or because it is the decision-makers who behave in a non-rational way (or both things). The fundamental difference between the decision theory and the game theory is that in the first, the decision-maker finds himself facing a decision-making problem within some "aleatory states of nature", of which he, maybe, holds a probabilistic characterization; in the second case the player is in front of another decision-maker as a competitor. As a consequence, while in a decision-problem the aim is to achieve an optimal choice (or a sequence of choices), in the second case it is necessary to elaborate a different concept, that is to say that one of equilibrium.

A first classification distinguishes games into *cooperative games* and *non-cooperative games.*

In *cooperative games*, the elementary unit is the coalition of players formed on the basis of binding pacts and opposed to other coalitions. The main problem that arises is not so much on the choice of the moves by the players, but rather on the way to distribute the higher profits from the partnerships.

In *non-cooperative games*, the elementary unit is the player opposed to the others. It is excluded that there can be binding preliminary negotiations, and collateral payments that can take place between the players. It means that each player gains what the outcome of the game attributes to him and that payments between players outside the game are not allowed. Game theory consists of three fundamental blocks:

1. *representation*: it consists of tools and methods to represent a situation of strategic interaction in a formal and graphic way;
2. *decision theory*, which is the representations of individual preferences;
3. *solution theory*: that is methods to understand how players behave.

In turn, the **representation** consists of the following elements:

a. a finite number of **players** $n \geq 2$;
b. the **moves** available to individual players and how they are made. If the players make their moves without knowing the choices made by the others

and the game ends after only one move (one shot game) we have static games. If some players, but not all, move first and the other players know the choices made by the first, we have dynamic games;
c. the **outcomes or payoffs** of the game which are the expected profits of the individual players;
d. **preferences** on outcomes.

In this paper we describe a soft approach to game theory proposed to high school students. From many points of view we have taken as a model the experimentation described in [6] concerning an educational approach to chaos theory, but obviously adapting the methodologies to our needs of a high school. In particular, our experimentation involved two classes of the last but one year at the IPSEOA "San Francesco" in Paola (CS), Italy. One has 22 students and the other 21, for a total of 43 students. We organized a cycle of 5 or 6 short lessons for a total of about 6-7 hours (depending on the class). The first part of the lesson concerned game theory: a general approach (a little more informal than our introduction above), the difference between strategic and extended forms, the concepts of Pareto optimal, Nash equilibrium and so on. Then, in a second smaller part of the cycle of lessons, we introduced the grossone-based numerical system (see Sect. 2 and Subsect. 2.1 in particular) and we presented to students how it is possible to work together with both of them.

A final class test was administrated to the students, whose results are discussed in details in Sect. 3. Here we report just one of the strongest final conclusion (see Sect. 4): the grossone system seems very appropriate to be taught in high schools, even in connection with (elementary) game theory.

2 Approaching elementary game theory and infinity computing in high schools

2.1 Game theory via extended and strategic forms

An elementary approach to game theory can be developed with the use of the *strategic form* or the *extended form*.

The representation in extended form takes place through a tree representation, while the strategic (or *normal*) form uses payoff matrices. Both forms have been proposed to students. Questions on the second form appeared in the final test (see Sect. 3), so we give here an example of the first approaches to the strategic form proposed to the students. It deals with the well-known game called *the battle of the sexes* that is explained below.

In separate places, Giulia and Marco choose to spend the evening at the cinema. Giulia prefers to go to the Odeon cinema because they're showing a comedy. Marco wants to go to the Luxe cinema because a thriller is being shown. Both Giulia and Marco would like to spend the evening together but Giulia prefers comedies while Marco prefers thrillers. In this case the players are two: Giulia and Marco; they make independent and simultaneous choices and have no information when they are making their choice. The moves available to the players are: 1) going to the Odeon cinema to see a comedy (C) or 2) going to the Luxe cinema to see a thriller (T); the outcomes are four: that is, all the possible combinations of the players' moves. The preferences (we indicate with the symbol "$>$" if an outcome is preferred to another) are given by the ordering on the outcomes.

For Giulia: "Odeon with Marco" $>$ "Luxe with Marco" $>$ "Odeon without Marco" $>$ "Luxe without Marco";

For Marco: "Luxe with Giulia" $>$ "Odeon with Giulia" $>$ "Luxe without Giulia" $>$ "Odeon without Giulia".

Giving the values $3 > 2 > 1 > 0$ to the order of the outcomes above, symmetrically for both the players Giulia and Marco, below there is the strategic representation of the game of sexes and its solution:

		Marco	
		C	T
Giulia	C	$(3, 2)$	$(1, 1)$
	T	$(0, 0)$	$(2, 3)$

C obviously stands for comedy, so at the Odeon cinema, T for thriller at the Lux. For instance, the entry at the bottom right, relating to the pair (2,3), is the best for Marco who obtains the maximum possible payoff equal to 3, while Giulia obtains 2. Note that there is not a best solution for both them simultaneously.

Similar examples, representations and exercises have been widely used in class with students. Through them, advanced concepts as *Pareto optimal point*, *Nash equilibrium*, iterated games, etc., have been proposed and explained to the students. Observations and conclusions on the students' response will be given in Sects. 3 and 4.

For general references on basic game theory the reader can see [7] or [23], instead for some examples of applications of game theory in education contexts he/she can see [8] and the references therein.

2.2 Infinity computing and game theory at school

Then it has been developed also a soft approach to infinity computing in the same two classes. Y. Sergeyev proposed a new numerical system at the beginning of this century: it allows to perform computations with infinite and infinitesimal numbers in an easy way, and it is constructed on a new fundamental infinite number called *grossone* and denoted by ①. We refer the reader to the introductory surveys [34, 37] or to the book [32]. In the last 20 years Sergeyev's system found a number of successfull applications in many areas of mathematics and also other sciences as physics, biology, etc. For instance, [1, 5, 17, 18, 22, 30, 37] contain applications of the grossone-based system to ordinary differential equations, cellular automata, game theory and optimization, [4, 9, 10, 15, 16, 29, 33, 36, 35] contain applications to fractals, summations and some problems concerning biology. The reader interested to deepen logic foundations of the new system and new solutions to old paradoxes, can see [11, 25, 37]. We deserve a special mention to a recent research line to which also the present paper belongs: the use of the grossone-based system in high schools for educational purposes (see [2, 3, 20, 21, 26–28, 31]).

Inside our cycle of class lessons, about 2 hours have been devoted to practical class with infinity computing. Such a short time was more than sufficient, by virtue of the great ease of use of the new system for basic calculations. Instead, most of the time was spent working on the connections between game theory and infinity computing. In particular, students met the ideas of sequence (finite or infinite) of games, called tournament, and the possibility to apply grossone-based computations on sequences of games. We will see some easy and basic examples in Sect. 3 discussing the final students test.

3 The final test

In this section we discuss the results obtained in the final test, after a short cycle of lessons of about 7-8 hours in total, divided in 5 or 6 days (depending on the class). As said in the previous section, about 2 hours have been used to teach to the students the very basic fundamentals of the grossone numerical system, also in connection with game theory. The interested reader who wants to learn more deep connections between the two fields can see [18, 19] and the references therein.

During the class lectures several examples of questions were proposed to the students, and in particular with open answers. The final students test instead consisted of 8 questions, the first 4 concerning only (basic) game theory and the last 4 dealing with connections between game theory and infinity computing. Furthermore, 7 questions had multiple predefined answers and one containing 5 sub-questions with true/false answers. The details for each question and the results of the students answers are below. The day of the final test 21 students were present in the first class and 19 in the second, for a total number of 40 students.

Question 1. Consider a game with two players (player1, player2), whose strategic representation is given by the following payoff matrix:

$$\begin{array}{ccc} & & \text{player2} \\ & & \begin{array}{cc} C & \quad T \end{array} \\ \text{player1} & \begin{array}{c} C \\ T \end{array} & \begin{array}{cc} (15, 15) & (5, 20) \\ (20, 5) & (10, 10) \end{array} \end{array} \tag{1}$$

If player1 makes the first move and chooses strategy C, which one is the best strategy for player2?

(a) Strategy T;
(b) Strategy C;

Table 1. Table of the answers for Question 1. The correct answer is obviously (a), strategy T.

Possible answers	Number of students	Percentage
(a) Strategy T	25	62.5%
(b) Strategy C	6	15%
(c) None of the two (T, C)	4	10%
(d) There is no single answer	5	12.5%
No answers	0	0%
Total	40	100%

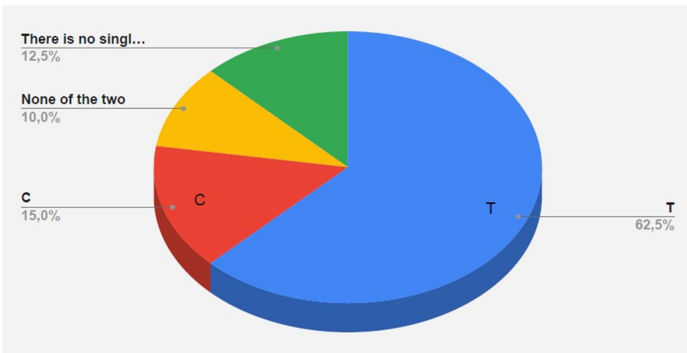

Fig. 1. A pie chart shows the percentages of student responses listed in Table 1 relatively to Question 1. The correct option is (a), strategy T, represented in blue.

(c) None of the two strategies T, C;
(d) There is no single answer.

Question 2. Consider the game of Question 1 with payoff matrix (1). Which one is the best strategy (for both) if the players can cooperate and then decide together the strategies to follow?

(a) (C, C): both players choose strategy C;
(b) (C, T): player1 chooses strategy C and player2 strategy T;
(c) (T, C): player1 chooses strategy T and player2 strategy C;
(d) (T, T): both players choose strategy T.

Question 3. Consider the game of Question 1 with payoff matrix (1). Which one is the best strategy (for both) if the players cannot cooperate and therefore they don't know each other's movements?

(a) (C, C): both players choose strategy C;
(b) (C, T): player1 chooses strategy C and player2 strategy T;
(c) (T, C): player1 chooses strategy T and player2 strategy C;
(d) (T, T): both players choose strategy T.

Table 2. Table of the answers for Question 2. The first option, (C, C), is the correct one.

Possible answers	Number of students	Percentage
(C, C): both players choose strategy C	19	47.5%
(C, T): player1 chooses strategy C and player2 T	5	12.5%
(T, C): player1 chooses strategy T and player2 C	9	22.5%
(T, T): both players choose strategy T	7	19.5%
No answers	0	0%
Total	40	100%

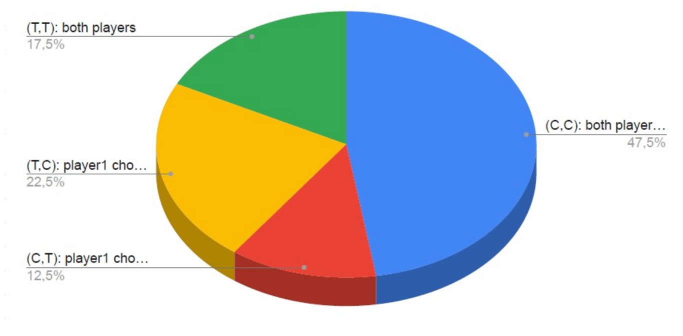

Fig. 2. A pie chart shows the percentages of student responses listed in Table 2 and relative to Question 2. The slice of the pie relating to the correct answer, (C, C), is colored blue.

Question 4. Consider again the game of Question 1 with payoff matrix (1). Players decide to play 5 times in sequence the game, i.e. a 5-game tournament. Initially, at the first two rounds, they cooperate in the choice of the strategy to pursue and they choose (C, C). But in the round 3, player2 decides to play T. Considering that the first three game strategies are (C, C), (C, C), (C, T), how will the evolution of the tournament be?

(a) 4th round (T, T) and 5th round (T, T);
(b) 4th round (C, T) and 5th round (C, T);
(c) 4th round (T, T) and 5th round (C, T);
(d) No definitive answer can be given.

Question 5. Choose true/false (T/F) for each of the following statements:

(a) A tournament can have 2① games;
(b) A tournament can have ① + 1 games;
(c) A tournament can have a number of games $n \leq$ ①;
(d) A tournament can only have a finite number of games;
(e) There can be no question of a tournament of ① games.

Table 3. Table of the answers for Question 3. The last option, (T, T), is the correct one.

Possible answers	Number of students	Percentage
(C, C): both players choose strategy C	8	20%
(C, T): player1 chooses strategy C and player2 T	9	22.5%
(T, C): player1 chooses strategy T and player2 C	10	25%
(T, T): both players choose strategy T	12	30%
No answers	1	2.5%
Total	40	100%

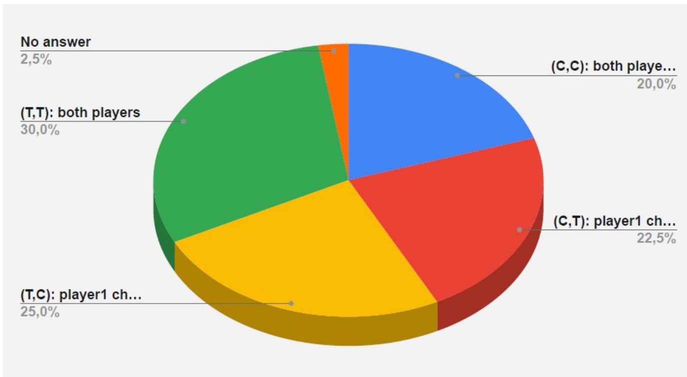

Fig. 3. A pie chart shows the percentages of student responses to Question 3. The green slice is related to the correct answer (T, T).

Question 6. Two players decide to play 2 tournaments, one after the other, each consisting of ① rounds. What can I say about the total number of games played?

(a) ① games;
(b) 2① games;
(c) Infinite games but we cannot say the number;
(d) 2① − 1 games;
(e) ① × ① games;
(f) No one can give a definite answer.

Question 7. Player1 and player2 play a series of games (possibly several consecutive tournaments). If one of the two players loses ① rounds he leaves the game and player3 enters in his place. Then player3 comes into play

(a) at the latest after 2① − 1 games;
(b) it does not make sense;
(c) after ① games;
(d) never.

Table 4. Table of the answers for Question 4. The first option, i.e. (a), is the correct one.

Possible answers	Number of students	Percentage
(a) 4th round (T, T) and 5th round (T, T)	14	35%
(b) 4th round (C, T) and 5th round (C, T)	9	22.5%
(c) 4th round (T, T) and 5th round (C, T)	10	25%
(d) No definitive answer can be given	7	17.5%
No answers	0	0%
Total	40	100%

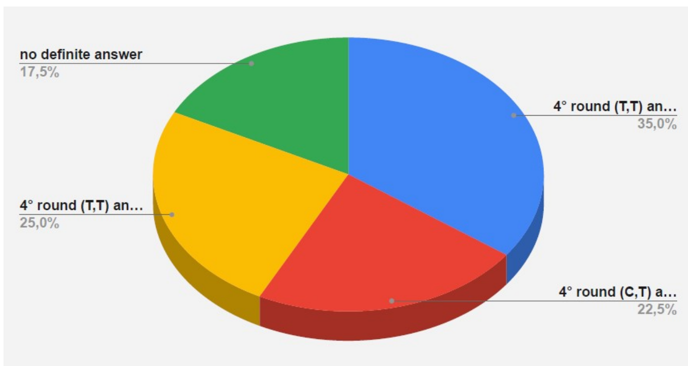

Fig. 4. A pie chart showing the percentages of student responses to Question 4. The blue slice is the one relating to the correct answer.

Question 8. Player1 and player2 play a series of games (possibly several consecutive tournaments). If one of the two players loses ①/3 rounds he leaves the game and player3 enters in his place. Then player3 comes into play

(a) after ①/3 games;
(b) after 2①/3 games;
(c) at the latest after $2①/3 - 1$ games;
(d) never.

4 Conclusions

The response of the students in terms of interest and participation was enthusiastic throughout the cycle of lessons. We also tried to make them play and have fun with the new concepts, exercises in class and more, with very good results and quite high approval from both classes.

The results of the final test have been in general very good. The most difficult point, for the students, was to manage the payoff matrix. It must be said that it is the first time they have seen a matrix in mathematics.

Table 5. Table of the answers for Question 5. The right answers are, in order, F, F, T, F, F.

Possible answers	T	% of T	F	% of F
(a) A tournament can have 2① games	10	25%	30	75%
(b) A tournament can have ① + 1 games	15	37.5%	25	62.5%
(c) A tournament can have a number of games $n \leq$ ①	29	72.5%	11	27.5%
(d) A tournament can only have a finite number of games	16	40%	24	60%
(e) There can be no question of a tournament of ① games	15	37.5%	25	62.5%

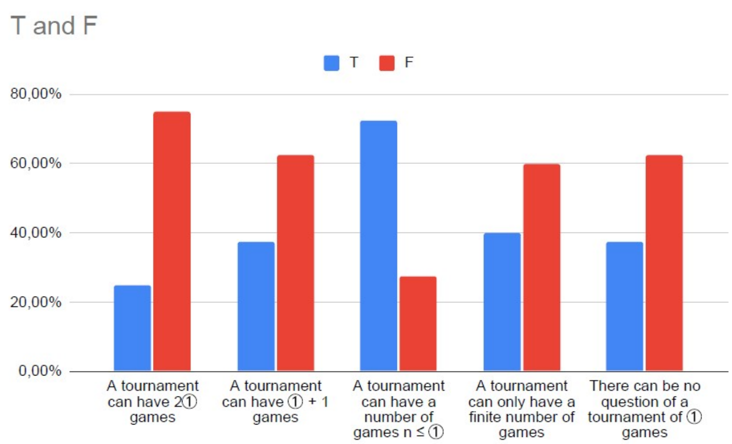

Fig. 5. A histogram showing the percentage of T/F answers for each of the 5 points that make up Question 5. The right answers are, in order, F, F, T, F, F.

Quickly analyzing the answers given by the students, we immediately notice how 62.5% answered the (easy) Question 1 correctly and 47.5% Question 2 concerning Pareto optimal. The percentage of correct answers drops to 30% for the more difficult Question 3 concerning Nash equilibrium and to 35% for Question 4.

The T/F answers of Question 5 were instead a great success, and this demonstrates once again the remarkable ease of use that students find handling the grossone system (cf. [2, 3, 21]).

A good performance also with Question 6, 40% of correct answers. The last two more difficult questions achieved more than acceptable percentages (also considering that the school in question is not a lyceum): 30% for Question 7 and 25% for Question 8.

The most surprising case, in our opinion, regards Question 5: for each of the 5 items (a)-(e), the majority of the students always gave the correct answers, and in particular from a minimum of 60% to a maximum of 75%. This leads us to believe, in agreement with the conclusions of [2, 3, 21], that the grossone-

Table 6. Table of the answers for Question 6. (b) is the correct one.

Possible answers	Number of students	Percentage
(a) ① games	6	15%
(b) 2① games	16	40%
(c) Infinite games but we can not say the number	7	17.5%
(d) 2① − 1 games	3	7.5%
(e) ① × ① games	4	10%
(f) No one can give a definite answer	4	10%
No answers	0	0%
Total	40	100%

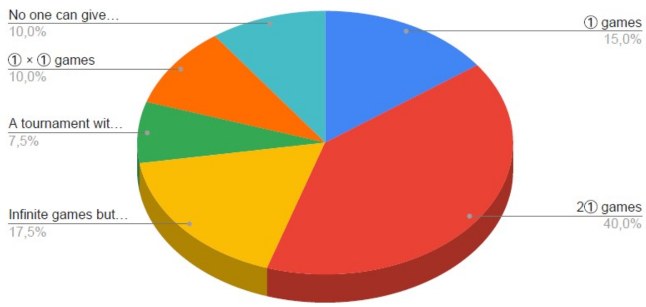

Fig. 6. A pie chart shows the percentages of student responses to Question 6. The red slice is the one relating to the correct answer.

based system can be taught with great ease and profit in high schools, also in connection with game theory.

In the near future we intend to carry out more extensive experiments involving a greater number of students, always in the same school or together with other schools. In particular we would like to broaden the experimental horizon in two main directions. In the first we intend to involve unimaginable numbers (see [14, 24] and the references within them for definitions and basic properties) which have many points of contact with game theory and the Infinity Computing (see also [2] in this same volume). A triptych formed in this way offers many ideas for work and didactic experimentation. The second direction instead moves towards geometry, and wants to experiment with Infinity Computing in connection with the succession of Carboncettus octagons, combinacorics of words (see [12, 13]), Fibonacci numbers and game theory. This direction, towards the combinatorics of (infinite) words and codes, also involves Infinity Computing from a purely theoretical point of view, posing many possible problems to investigate.

Table 7. Table of the answers for Question 7. The correct answer is (a).

Possible answers	Number of students	Percentage
(a) At the latest after 2① − 1 games	12	30%
(b) It does not make sense	10	25%
(c) After ① games	10	25%
(d) Never	8	20%
No answers	0	0%
Total	40	100%

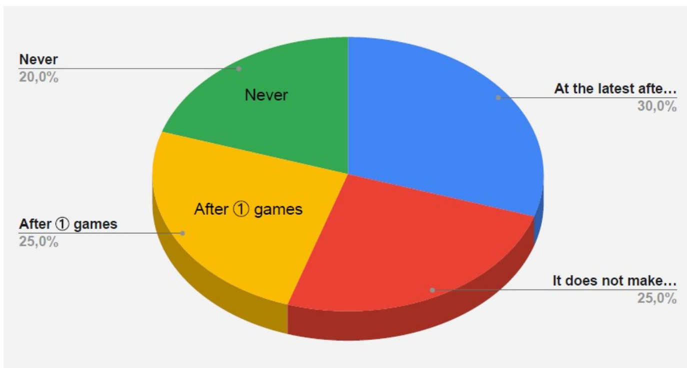

Fig. 7. A pie chart showing the percentages of student responses to Question 7. The blue slice is the one relating to the correct answer (a).

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Table 8. Table of the answers for Question 8. The correct option is (c).

Possible answers	Number of students	Percentage
(a) After ①/3 games	15	37.5%
(b) After 2①/3 games	12	30%
(c) At the latest after 2①/3 − 1 games	10	25%
(d) Never	3	7.5%
No answers	0	0%
Total	40	100%

Fig. 8. A pie chart showing the percentages of student responses to Question 8. The yellow slice is the one relating to the correct answer (c).

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